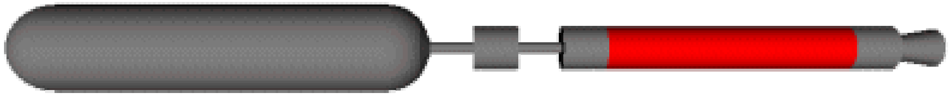
AE6450 Fall 2004
Lecture #12
Hybrid Rocket Engines

Hybrid Rocket Motors

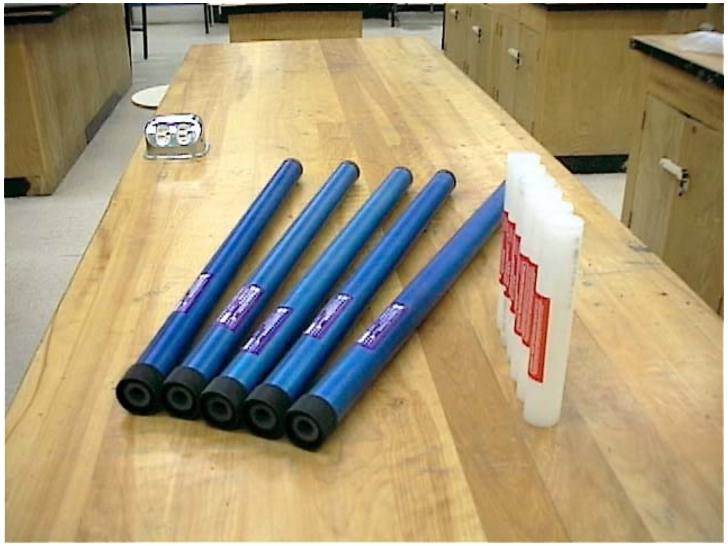
Hybrid Rocket Motors
"Hybrid Rockets" combine one liquid propellant with one solid propellant – typically a solid fuel and a liquid oxidizer.



www.ukrocketman.com/rocketry/ hybridscience.shtml

Oxidizers
- LOX
- GOX - common
- H_2O_2
- N_2O_4
- LF_2 (Can be combined with LOX)

Hybrid Rocket Motors



www.propulsionpolymers.com/ images/MarcusBill.jpg

Hybrid Rockets

- Safer than liquids and pre-mixed solids (easy handling)
- Can be throttle, shut down
- I_{sp} is between solids and liquids (300-320 sec)
- Bulk density is good (better than liquid, but not quite as good as a solid)
- Do not have toxic exhaust like - based solids
- Not as sensitive to exposed cracks like solids
- Low regression rate (needs more port area to generate required thrust)
- Burn is often incomplete due to complex port design – shows combined fuel one left at the end of the burn (lower effective mass fraction)

Hybrid Rocket Motors

Fuels

HTBP: Hydroxy-terminated polybutadene: rubbery HC binder from solid fuels
Plexiglass (Polymethyl-methacrylate PMM)

(Early: coal, wood, rubber)

In some cases carbon black or aluminium powder can be added to the fuel grain to increase burning

Burn rate – Regression rate

There are several burn rate models, but a good one is: $r = aG^n X^m$

G = total mass flux ($\frac{Kg}{m^2 - S}$ **) (fuel+oxidizer)**

X = distance down the port (m)

a = regression coefficient

In theory,

$n \approx .80$

$m \approx -.20$

$a \approx 2.066 * 10^{-5} m/s$

However, experimentally

$$n \approx .75 - .77$$

$$m \approx -.14 - .16$$

Note: This equation uses $G_f(x) + G_0(x) = G$

where both are a function of $r(x)$

so the implicit solution is iterative to

determine $G_f(x)$

Burn rate depends on flow rate and downstream distance

Here, the key observations are,

- **r is proportional to G(x) (and thus G) (can be turned off)**
- **burn rate decreases with length along L.**

This is primarily due to the diffusion nature of the flame region. As the boundary layer grows, flame moves away from surface; heating rate decreases.

As a result, the regression is slightly longer at the injector end

Other Empirical Fits For Burn Rate

Other burning rate equations exist and may be better in some cases; e.g.,

$$r = aG_0{}^n L^m$$ **(averaged method)**

r **average regression rate**

G_0 **average oxidizer flux**

L **port length**

This equation is simpler in that it depends on average values. May be ok for performance predictions.

Show table 7.5 from Humble – curve fits to AMROC data for various equations.

Change in O/F Ratio

As the solid fuel is burned (web is consumed) the port area increases and thus G_{ox} decreases .

As such, $\dot{m}_f$ **decreases and** $\frac{O}{F} = \frac{\dot{m}_{ox}}{\dot{m}_f}$ **will increase over time.**

This causes a shift in

$\gamma, C^*,$ **and** I_{sp} **as the motor burns. Over a given O/F range the** I_{sp} **might shift by a few %.**

Typical Port Configurations

In general, two ports will have the same oxidizer flow rates if their hydraulic diameters are equal.

Objective: maximize volumetric efficiency, minimize sliver fraction,

keep $\dot{m}_{ox}$ **(thus G_{ox}) the same in all ports.**

Fig. 7.17&18 from Humble shows typical port configurations for hybrid rockets, and the associated variations of volumetric efficiency and chamber length-to-diameter ratio (L/D) as a function of required fuel mass.

- **Circular : Low** η_{vg} **and large size.**
- **7-cylinder cluster : high sliver fraction (residuals)**
- **Wagon Wheel with center port: good compromise at larger size**

Note: The wagon wheel design is more difficult to solve analytically, but can be solved numerically.

Port Configuration

Using multiple ports (holes) on a hybrid system will allow us to have more surface area exposed to the oxidizer and thus generate more thrust for a given diameter.

Define Volumetric efficiency

$$\eta_{vg} = \frac{VolumeOccupiedBySolidFuel}{VolumeInsideCase(OfThe"Grain")}$$

"Skew Fraction" = (1- mass of solid fuel used) / (mass of solid fuel available)

Hydraulic diameter =

$$D_n = \frac{4*(CrossSectionalAreaOfAPort)}{PerimeterOfAPort}$$

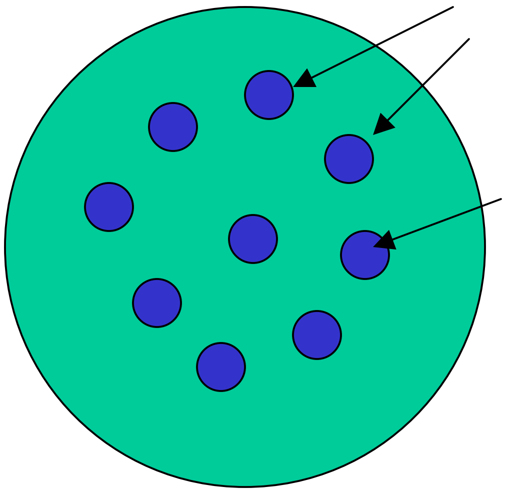
Note: for a circular port, D_n **=D**

$$D_n = 4\left[\frac{\pi\left(\frac{D}{2}\right)^2}{\pi D}\right] = D$$

Performance Analysis

For a given propellant with: $\dot{r}_{avg} = aG_{ox}{}^{n}L^{m}$ **m/s regression rate**

and a given grain geometry



For a given $\dot{m}_{ox}$ **, find** $G_{ox} = \frac{\dot{m}_{ox}}{N\Delta P}$

$\dot{r} = aG_{ox}{}^{n}L^{m}$ **alternate versions of this are available**

(e.g., $f_{(G_{Total})}$ **but require iteration)**

Dimensions known: D_n and L given $\dot{m}_f = \left(\rho_g S_P N\right)\dot{r}$ **N= number of ports**

$\dot{m}_p = \dot{m}_{ox} + \dot{m}_{fuel}$ **ignore gas added to chamber**

For a given $\dot{m}_{ox}$, find $G_{ox} = \frac{\dot{m}_{ox}}{NA_P}$

$\dot{r} = aG_{ox}{}^{n}L^{m}$ alternate versions of this are available

(e.g., $f_{(G_{Total})}$ but require iteration)

$$\dot{m}_f = \left(\rho_g S_P N\right)\dot{r}$$

N= number of ports

Dimensions known: Dn and L given

(density of solid) (surface area of a single port)
(assume identical here)

$$\dot{m}_p = \dot{m}_{ox} + \dot{m}_{fuel}$$

ignore gas added to chamber

Time-stepping procedure

$\frac{O}{F}$ **is** $\frac{\dot{m}_{ox}}{\dot{m}_{fuel}} \Rightarrow$ **find** γ, C^*, T_f, η

from charts in Appendix B of Humble

Since $$\frac{C^* C_F}{g_0} = I_{sp} = \frac{T}{\dot{m}_{prep} g_0} = \frac{C_F P_c A_t}{\dot{m}_{prep} g_0}$$

$$P_c = \frac{\dot{m}_{pop} C^*}{\Delta t}$$

Given $\mathcal{E}$ find CF and thus thrust and Isp (Note: throat erosion can cause A_t to change over time)

For a small time step, Δt hold $\dot{r}$ constant and use

$\Delta r = \dot{r}\,\Delta t$ to find new port cross section area (A_p) and surface area S_p

Repeat until first web segment is nearly consumed or tb is reached.

Example calculation from pp 433-434 in Humble

I_{sp} vs. time from table

Thrust vs. time from chart

(7 port Wagon Wheel, no burning in center port)

Hybrid Motor Ballistics

This derivation is from the text by Humble. It is presented to illustrate the basis for the empirical expressions for burn rate. Equation numbers are the same as those in Humble et al.

The burn rate of a solid fuel in a hybrid rocket motor also depends on oxidizer flow rate. Thus fuel regression rate is

$$\dot{r} = aG^{n}x^{m} \ldots\ldots\ldots 7.1$$

Here,

$\dot{r}$ **is expressed in m/s. Note that its values are typically in mm/s or cm/s.**
G= total propellant in Kg/m2s
X= distance down the point
A, n, m are regression rate constants of the propellant.

$\dot{r}$ is a function $f\left(\dot{m}\right)$;

$\dot{r}$ is also $g\left(x\right)$

G increases with x.

A major difference from solid and liquid rockets: hybrids burn in a diffusion flame, as opposed to the premixed flames in the other types of rockets. In solid rockets, O/F is independent of x. However, in hybrids the O/F ratio does vary with x.

Interior Ballistics Model

Turbulent diffusion flame
Controlling factors:

- **rate of heat transfer to surface.**
- **heat of decomposition of solid fuel.**
- **G determines rate of heat generation and hence flow**

The flame occurs within the turbulent boundary layer. Please refer to the figure showing the boundary layer and the flame zone inside the boundary layer. Note that the axial velocity (i.e., the velocity component directed along the axis of the port) at the flame zone is thus lower than the velocity at the edge of the boundary layer.

$$U_b < U_e$$

The burning rate equation is:

$$\dot{Q}_w = \dot{m}_f \, h_v \ldots\ldots\ldots(7.2)$$

$\dot{Q}_w =$ heat flux transferred from flame to solid surface $J/m^2 s$

$\dot{m}_f =$ mass flux rate of vaporized fuel, perpendicular to surface $Kg/m^2 s$

$h_v =$ heat content of unit mass of gasified fuel at surface minus heat content of solid at ambient temperature (J/Kg)

includes:

1. heat to warm solid to surface temperature
2. thermal changes such as depolymerization
3. heat of vaporization

h_v measured in labs.

Heat transfer through boundary layer occurs by conduction, which is proportional to the thermal gradient.

$$\dot{Q}_w = -k\frac{\partial T}{\partial y} = \frac{-k}{C_p}\frac{\partial h}{\partial y} \quad \ldots\ldots\ldots(7.3)$$

k: molecular/turbulent boundary layer gas conductivity (J/msk)
T: gas temperature at any point y(x)
h: Specific enthalpy of gas at any point y (J/kg)

C_p : **Specific heat of gas at constant pressure (J/kg)**

The form in terms of the enthalpy gradient:

$$\dot{Q}_w = -k\frac{\partial h}{\partial y}$$

is useful when chemical recombination occurs in the boundary layer.

In terms of the Stanton # C_H

$$\dot{Q}_w = C_H \rho_b U_b \Delta h$$

$\rho_b U_b$: axial mass flux in flame zone $(kg/m^2 s)$

Δh : Total specific enthalpy difference between flame and wall (J/kg)

ρ_b : gas density at flame $\left(kg/m^3\right)$

U_b : Gas velocity at gas flame (m/s)

CH can be determined from friction coefficient CF for which experimental data are available for turbulent flow over flat plate.
Assume:

- **Prandtl # = 1 =** $\frac{C_p \mu}{k}$

- **Lewis # = 1 (diffusivity of heat and molecular species are equal)**

Reynolds analogy: temperature/enthalpy profile $\frac{dh}{dy}$

through a boundary layer is

proportional to $\frac{dU}{dy}$

Note: Non-dimensional parameters (source: Hill & Peterson)

Nusselt number (for convective heat transfer):

$$N_u = \frac{h_f x}{k} = \frac{1}{2}\mathrm{Re}_x C_f$$

Stanton # $St = C_H = \dfrac{h_f}{\rho C_p U}$

Heat transfer Film coefficient: h_f : **is such that**

$\dot{Q} = h_f\left(T_w - T_\infty\right)$ **. From this, we can see that**

$$h_f = \frac{\dot{Q}}{\Delta T}$$

and

$$\frac{\dot{Q}}{h_f} = \Delta T$$

Using Reynolds analogy,

$$\frac{\dot{Q}}{\dot{Q}_w} = \frac{\tau}{\tau_w} \text{.........}(7.6)$$

τ : **shear stress at y (Pascals)**

U: axial velocity at y

$$\frac{\dot{Q}}{\partial h / \partial y} = \frac{\tau}{\partial U / \partial y} \text{.........}(7.5)$$

Integrating equation 7.5 between the burning zone and the surface,

$$\frac{\dot{Q}_w}{\Delta h} = \frac{\tau_w}{U_b} \ldots\ldots\ldots\ldots\ldots(7.7)$$

Divide by $(\rho_b U_b)$;

Compare with 7.4:

$$C_H = \frac{\dot{Q}_w}{\rho_b U_b \Delta h} = \frac{\tau_w}{\rho_b {U_b}^2} \ldots\ldots\ldots(7.8)$$

From definition of C_f :

$$\tau_w = \frac{1}{2}\rho_e U_e{}^2 C_f \ldots\ldots\ldots\ldots\ldots(7.9)$$

where

ρ_e = freestream gas density (kg/m²)

U_e = freestream m/s

C_f = flow friction coefficient

From (7.8)

$$C_H = \frac{C_f}{2} \frac{\rho_e U_e^{\,2}}{\rho_b U_b^{\,2}} \ldots\ldots\ldots\ldots\ldots\ldots (7.10)$$

For non-combusting boundary layer,

$$C_{H_0} = \frac{C_{f_0}}{2} \ldots\ldots\ldots\ldots\ldots\ldots (7.11)$$

Here, boundary layer is extended due to "blowing" from the surface due to fuel vaporizing. Expecting blowing to have similar effects on heat and momentum transfer

$$\frac{C_H}{C_{H_0}} = \frac{C_f}{C_{f0}} \quad \text{...............} (7.12)$$

Combine 7.2, 7.4, 7.12, 7.13: Basic expression for burning rate in hybrid rocket:

$$\dot{m}_f = \dot{r}\,\rho_f = 0.03\left(\rho_e U_e\right) R_e^{-0.2} \left[\frac{C_H}{C_{H_0}}\right]\left[\frac{U_e}{U_b}\right]\left(\frac{\Delta h}{h_v}\right) \quad \text{.....} (7.14)$$

Evaluate skin friction coefficient C_f

from empirical law for turbulent boundary layers.

Empirical law for $P_r = 1.$ **is**

$$\frac{C_{f0}}{2} = C R_e^{-0.2}$$

C: 0.3 (empirical)

$$R_e = \frac{G_x}{\mu} \quad \text{............} (7.15)$$

With equation 7.15 for R_e

$$\dot{m}_f = \dot{r}\,\rho_f = \frac{0.3\mu^{0.2}}{x^{0.2}} G^{0.8} \left[\frac{C_H}{C_{H_0}}\right]\left[\frac{U_e}{U_b}\right]\left(\frac{\Delta h}{h_v}\right) \quad .....(7.16)$$

$\dot{m}_f$: mass flux rate of fuel from solid surface (kg/m²s)

$\dot{r}$ = regression rate (m/s) of solid fuel.

ρ_f = density of solid fuel

μ : viscosity of burned gas

C_H : Stanton # with blowing

C_{H_0} : Stanton # without blowing

Combine Eqns. (2) (4), (10)

$$B = \left(\frac{U_e}{U_b}\right)\left(\frac{\Delta h}{h_v}\right) \ldots\ldots\ldots\ldots\ldots\ldots (7.18)$$

(determined from propellant properties.)
Maxman 1964:

$$\frac{C_f}{C_0} = \left(\frac{\ln(1+B)}{B}\right)^{0.8} \left[\frac{1+1.3B+0.364B^2}{\left(1+\frac{B}{2}\right)^2 (1+B)}\right] \ldots\ldots\ldots\ldots\ldots (7.19)$$

or

$$\frac{C_f}{C_{f0}} = 1.2B^{-0.77} \text{............}(7.20)$$

In the range 5 < B < 20 (usual in hybrid systems)

$$\frac{C_f}{C_{f0}} = B^{-0.68} \text{............}(21)$$

Final expression:

$$\dot{M}_f = \dot{r}\rho = 0.03\left(\frac{\mu}{x}\right)^{0.2} G^{0.8}B^{0.32} \text{.............}(22)$$

Contrast to solid rocket $r = ap_e{}^n$

Equation (22) is:

$$\dot{r} = aG^n x^m$$

Dependence on axial length

$$G(x) = G_0 + G_f(x)\dots\dots\dots\dots\dots(7.25)$$

G_0 : oxidizer mass flux in port (kg/ms)
$G_f(x)$: fuel flux added to port in front of point x (kg/m^{2}s)

Regression rate

$$\dot{r}(x) = ax^m \left[G_0 + G_f(x)\right]^n \qquad (7.26)$$

where $G_f(x)$

is determined by integrating fuel-mass addition along the port:

$$G_f(x) = 4\rho \int_0^x \frac{\dot{r}(x)}{D_H} dx \qquad (7.27)$$

D_H : hydraulic diameter (m) = 4A/P

A: port C/S area (m^2)

P: port perimeter (m)

Data shows that hybrid combustion down the port provides fairly constant burn rates:

Why

1. Boundary layer grows as x^m -> decreases heat flux balanced by increase in G due to adding fuel.
2. Spurious higher local C/S causes reduced local mass flux -> levels out contours.

Combustion instability in hybrids

Hybrid rockets often exhibit combustion instability (pressure oscillations) during burning derived from

1. oxidizer feed pressure oscillations (Chugging)
2. flame holding instability (acoustic)

Increasing the injection velocity and reducing sources of compressibility in the feed system will address the former (#1).

Preheating the oxidizer either with a hydrogen or propane pilot flame or by creating a re-circulation zone at the head-end of the motor will improve the latter (#2)

(Fig. 15-13 Sutton)